Agent-based Computational Economics. A Short Introduction.

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Agent-based computational economics: a short introduction

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Abstract

In this paper we provide a brief overview of the main characteristics of agent-based computational economics. We discuss its points of strength, with respect to analytical models, and its weaknesses. The latter are mainly related to how the results of a simulation model can be interpreted, and how the structural parameters of the model can be estimated. We then show how these problems can be dealt with.

1 Introduction

In a nutshell, agent-based models (ABMs) are models, that is, abstract representations of the reality, in which (i) a multitude of objects interact with each other and with the environment, (ii) the objects are autonomous, that is, there is no central, or 'top down' control over their behavior[1], and (iii) the outcome of their interaction is numerically computed. Since the objects are autonomous, they are called 'agents'. The application of agent-based modeling to economics has been labeled agent-based computational economics (ACE). As Leigh Tesfatsion—one of the leading researchers in the field and the 'mother' of the ACE acronym—defines it, ACE is

> the computational study of economic processes modeled as dynamic systems of interacting agents (Tesfatsion, 2006).

Note that none of the two features above, in isolation, defines the methodology: the micro-perspective implied by (i) and (ii) is the same adopted, for instance, by game theory, where strategic interaction is investigated analytically, while the computational approach is typical of Computational General Equilibrium or System Dynamics, which, however, are based on aggregate representations of the system.

In this paper we will describe in more detail the features of ABMs (Section 2), offering an overview of their historical development (Section 3), discussing when they can be fruitfully employed, and how they can be combined with more traditional approaches (Section 4). While maintaining a 'low profile' in describing the approach, we will offer a strong defense of its methodological soundness (Section 5). In particular, we will argue that (i) ABMs are mathematical models, (ii) ABMs may lead—as do analytical models—to general results, and (iii) ABMs can be taken to the data, that is, estimated empirically. Section 6 concludes. In this paper we will not discuss the issue of validation of ABMs, the main reason being that many problems in validation are the same encountered with more traditional (*analytic*) models. The interested reader is referred

[1] As is the case of the Walrasian auctioneer device for ensuring market clearing, for instance. More on this point in Section 3 below.

to the special number of *Computational Economics* explicitly devoted to empirical validation of ABMs, and in particular to Fagiolo *et al.* (2007) and Marks (2007)[2].

2 Features of agent-based models

The basic units of ABMs are 'agents'. Agents can be anything from cells to biological entities, from individuals to social groups such as families or firms. Agents can be composed by other agents: the only requirement being that they are perceived as a unit from the outside, and that they 'do' something, that is, they have the ability to *act*, and possibly to *react* to external stimuli and *interact* with the environment and other agents.

The environment, which may include physical entities (such as infrastructures, geographical locations, etc.) and institutions (such as markets, regulatory systems, etc.) can also be modeled in terms of agents (e.g. a central bank, the order book of a stock exchange, etc.), whenever the conditions outlined above are met. When not, it should be thought of simply as a set of variables (say 'temperature' or 'business confidence').

From what we have said so far, it should be clear that aggregate variables like Consumption, Savings, Investments, Disposable Income, etc., which are the prime units of analysis of Keynesian macroeconomics, cannot be modeled as such in an agent-based framework—they have to be (easily) computed by aggregating microeconomic agent quantities; nor there is room for the fictitious representation of a 'Representative Agent', a cornerstone of neoclassical economics. Therefore, ACE can in principle provide sound microfoundations for the 'traditional' Keynesian macroeconomics, and sound aggregate results for the neoclassical analysis based on individual optimization. The direct modeling of a demand or a supply curve is also forbidden in an agent-based setting: rather, these aggregate functions might (or might not) emerge as the outcome of the decisions of the individual agents.

2.1 The whole and its parts

Having agents as the unit of analysis, agent-based modeling is deeply rooted in *methodological individualism*. This doctrine was introduced as a methodological precept for the social sciences by Max Weber, most importantly in the first chapter of *Economy and Society* (Weber, 1922 (1968))—although the term was already present in Schumpeter (1909). It amounts to the claim that social phenomena must be explained by showing how they result from individual actions, which in turn must be explained through reference to the intentional states that motivate the individual actors[3]. However, the term still bears some ambivalence over whether explanations should be in terms of individuals alone, or in terms of individuals plus their interactions (Hodgson, 2007). In the first meaning, methodological individualism suggests that the 'whole' is nothing but the 'sum of its parts', a position that has been labeled *reductionism* (Jones, 2000). This interpretation implies that the aggregate behavior can be derived observing the behavior of a single agent, a position that is clearly incompatible with the agent-based modeling approach. On the other hand, reductionism is implicit in the Representative Agent paradigm, which claims that the whole of society can be analyzed in terms of the behavior of a single, representative, individual.

The opposite view is *holism*, the idea that the properties of a given system cannot be determined or explained by the sum of its component parts alone. Instead, the system as a whole determines in an important way how the parts behave[4]. As such, holism is closely related to *organicism*, introduced as

[2] See also Leombruni *et al.* (2005).

[3] The use of methodological individualism in Economics was championed by the Austrian School of Economics in the XX century, of which Friederich von Hayek was one of the main exponents (von Hayek, 1948). The legacy of Hayek to agent-based modeling and the complex system approach (see e.g. von Hayek, 1967) has been amply recognized (Rosser, 1999; Vaughn, 1999; Koppl, 2000; Vriend, 2002; Rosser, 2009).

[4] The general principle of holism was concisely summarized by Aristotle in his Metaphysics: 'The whole is more than the sum of its parts'.

a biological doctrine stressing the importance of the organization, rather than the composition, of organisms[5]. This view has gained renewed popularity as a new science of Complexity—which, as we will discuss in the next section, is to a large extent responsible for the introduction of ABMs in the study of social and biological systems—developed in the last decades of the XX century.

So, where does agent-based modeling stand in this debate? As already noted, ABMs are characterized by the fact that aggregate outcomes (the 'whole') are computed as the sum of individual characteristics (its 'parts'). However, aggregate behavior can often be recognized as distinct from the behavior of the comprising agents, leading to the discovery of unexpected ('emergent') properties. In this sense, the whole is more than—and different from—the sum of its parts. As the Nobel price-winner phisicists Philiph Anderson concisely expressed it, 'more is different' (Anderson, 1972). It might even be the case than the whole appears to act *as if* it followed a distinct logic, with own goals and means, as in the example of a cartel of firms that act in order to influence the market price of a good. From the outside, the 'whole' appears no different from a new agent type. A new entity is born, the computational experiment has been successful in 'growing artificial societies from the bottom up'[6].

2.2 The dual problem of the micro–macro relation

As we have seen, agent-based modeling allows us to investigate the interplay occurring at two different scales of a given system: the microstructure and the macrostructure. This investigation may occur in two directions: (i) to find the aggregate implications of given individual behaviors and (ii) to find the conditions at the microlevel that give rise to some observed macrophenomena. We will refer to these two perspectives as the *dual problem* of the micro–macro relation. Both share the same approach: *If you didn't grow it, you didn't explain it* (Epstein, 1999), which motivates the definition of ACE as *generative social science*.

Of course, ABMs are by no means the only way to study the dual problem of the micro–macro relation. However, taking into account the interaction of a multitude of (possibly heterogeneous) agents, of possibly different types, easily becomes analytically intractable, and the traditional approach of simplifying everything may—as it should be clear from the discussion above—'throw the baby out with the bath water'. On the contrary, ABMs only require us to 'wait and see' the unveiling of the consequences of the assumptions, and leave much more freedom than conventional economics in the specifications of the assumptions.

2.3 Additional features of agent-based models

We have so far introduced the three fundamental characteristics of ABMs: there are agents that play the role of actors, there is no script or *Deus ex-machina*[7] and the story is played 'live', that is, computed.

However, there are a number of characteristics that are often found in ABMs, and may motivate their use. Following Epstein (1999, 2006), we can include:

- *Heterogeneity*. While in analytical models there is a big advantage in reducing the ways in which individuals differ, the computational burden of an ABM does not change at all if different values of the parameters (e.g. preferences, endowments, location, social contacts, abilities, etc.) are specified for different individuals. Normally, this is done by choosing a distribution for each relevant parameter, and this simply implies that a few parameters (those governing the distribution) are added to the model.

[5] William Emerson Ritter coined the term in 1919.

[6] As in the title of the well-known book by Joshua Epstein and Robert Axtell (Epstein & Axtell, 1996).

[7] In the Greek theater, a mechanism was used to drop one or more divinities on the stage to solve complicated situations, in which no apparent ways out were available.

- *Explicit space*. This can be seen as specification of the previous point: individuals often differ in the physical place where they are located, and/or in the neighbors with whom they can or have to interact (which define the network structure of the model).
- *Local interaction*. Again, this can be seen as a specification of the network structure connecting the agents. Analytical models often assume either global interaction (as in Walrasian markets), or very simple local interaction. ABMs allow for much richer specifications.
- *Bounded rationality*. Interestingly, while in analytical models it is generally easier to implement some form of optimal behavior rather than solving models where individuals follow 'reasonable' rules of thumb, or learn either by looking at what happened to others or what happened to themselves in the past, for ABMs the opposite is true (Edmonds, 1999; Manson, 2006; Pyka & Fagiolo, 2007). However, it can be argued that *real* individuals also face the same difficulties in determining and following the optimal behavior, and are characterized by some sort of bounded rationality (Conlisk, 1996; Gigerenzer & Selten, 2001). To quote Epstein,

  > There are two components of this: bounded information and bounded computing power. Agents have neither global information nor infinite computational capacity. Although they are typically purposive, they are not global optimizers; they use simple rules based on local information (Epstein, 2006).

- *Non-equilibrium dynamics*. ABMs are recursive models, in which the state of the system at time $t + 1$ is computed starting from the state at time t. Hence, they allow the investigation of what happens all along the route, not only at the start and at the end of the journey.

The latter point is, we believe, the most important. W. Brian Arthur offered a beautiful and concise statement of its relevance for economic theory:

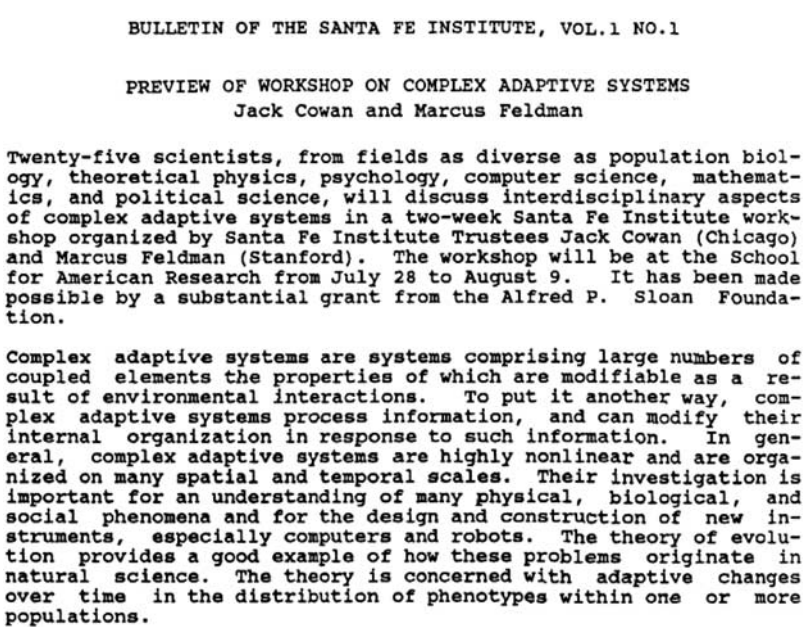
> Standard neoclassical economics asks what agents' actions, strategies, or expectations are in equilibrium with (consistent with) the outcome or pattern these behaviors aggregatively create. Agent-based computational economics enables us to ask a wider question: how agents' actions, strategies or expectations might react to—might endogenously change with—the pattern they create. In other words, it enables us to examine how the economy behaves out of equilibrium, when it is not at a steady state.
>
> This out-of-equilibrium approach is not a minor adjunct to standard economic theory; it is economics done in a more general way. […] The static equilibrium approach suffers two characteristic indeterminacies: it cannot easily resolve among multiple equilibria; nor can it easily model individuals' choices of expectations. Both problems are ones of formation (of an equilibrium and of an 'ecology' of expectations, respectively), and when analyzed in formation—that is, out of equilibrium—these anomalies disappear (Arthur, 2006).

3 The development of agent-based computational economics

3.1 The Santa Fe perspective: the economy as an evolving complex system

The development of ACE is closely linked with the work conducted at the Santa Fe Institute, a private, not-for-profit, independent research and education center founded in 1984 in Santa Fe, New Mexico. The purpose of the Institute has been, since its foundation, to 'foster multi-disciplinary collaboration in pursuit of understanding the common themes that arise in natural, artificial, and social systems'. This unified view is the dominant theme of what has been called the new *science of complexity*[8].

[8] See also, among many others, Edmonds (1999), Phelan (2001) and Chu *et al.* (2003) and especially the popular books by Gleick (1987) and Waldrop (1992). A rather critical view of the research on complex systems undertaken at the Santa Fe Institute through the mid-1990s can be found in the writings of the



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PREVIEW OF WORKSHOP ON COMPLEX ADAPTIVE SYSTEMS
Jack Cowan and Marcus Feldman

Twenty-five scientists, from fields as diverse as population biology, theoretical physics, psychology, computer science, mathematics, and political science, will discuss interdisciplinary aspects of complex adaptive systems in a two-week Santa Fe Institute workshop organized by Santa Fe Institute Trustees Jack Cowan (Chicago) and Marcus Feldman (Stanford). The workshop will be at the School for American Research from July 28 to August 9. It has been made possible by a substantial grant from the Alfred P. Sloan Foundation.

Complex adaptive systems are systems comprising large numbers of coupled elements the properties of which are modifiable as a result of environmental interactions. To put it another way, complex adaptive systems process information, and can modify their internal organization in response to such information. In general, complex adaptive systems are highly nonlinear and are organized on many spatial and temporal scales. Their investigation is important for an understanding of many physical, biological, and social phenomena and for the design and construction of new instruments, especially computers and robots. The theory of evolution provides a good example of how these problems originate in natural science. The theory is concerned with adaptive changes over time in the distribution of phenotypes within one or more populations.

Figure 1 Excerpt from the Bulletin of the Santa Fe Institute, Vol. 1, No. 1, June 1986

For what concerns economics, the main outcomes of the research project conducted at the Santa Fe Institute (Figure 1) were three books, all bearing the title *The economy as an evolving complex system* (Anderson *et al*., 1988; Arthur *et al*., 1997; Blume & Durlauf, 2006). From the preface of the 1997 volume, edited by W. Brian Arthur, Steven Durlauf and David Lane,

> In September 1987 twenty people came together at the Santa Fe Institute to talk about 'the economy as a evolving, complex system'. Ten were theoretical economists, invited by Kenneth J. Arrow, and ten were physicists, biologists and computer scientists, invited by Philip W. Anderson. The meeting was motivated by the hope that new ideas bubbling in the natural sciences, loosely tied together under the rubric of 'the sciences of complexity', might stimulate new ways of thinking about economic problems.
>
> […] But just what is the complexity perspective in economics? That is not an easy question to answer. […] Looking back over the developments in the past decade, and of the papers produced by the program, we believe that a coherent perspective—sometimes called the 'Santa Fe approach'—has emerged within economics (Arthur *et al*., 1997).

Arthur goes on in describing the main characteristics of the Santa Fe approach[9]. These were identified in models having *cognitive foundations*, *structural foundations*, *no global controller*, and exhibiting *continual adaptation*, *perpetual novelty* and *out-of-equilibrium dynamics* (Arthur, 1990).

Ten years and two volumes later, Blume and Durlauf summarize this intellectual Odyssey as follows:

> On some levels, there has been great success. Much of the original motivation […] revolved around the belief that economic research could benefit from an injection of new mathematical

(F'note continued)
science journalist John Horgan (Horgan, 1995, 1997). A very good account of the relationships between complexity theory, cybernetics, catastrophe theory and chaos theory (the four 'C's) and their implications for economic theory, can be found in Rosser (1999).

[9] Although this perspective is associated with the Santa Fe Institute, it was initiated in Europe by chemists and physicists concerned with emergent structures and disequilibrium dynamics (more precisely, in Brussel by the group of the Nobel-prize-winning physical chemist Ilya Progogine and in Stuttgart by the group of the theoretical physicist Hermann Haken)—see Haken (1983), Nicolis and Prigogine (1989) and Prigogine and Stengers (1984).

> models and new substantive perspectives on human behavior. [...] At the same time [...] some of the early aspirations were not met. The models presented here do not represent any sort of rejection of neoclassical economics. One reason for this is related to the misunderstanding of many non-economists about the nature of economic theory; simply put, the theory was able to absorb SFI-type advances without changing its fundamental nature. Put differently, economic theory has an immense number of strengths that have been complemented and thereby enriched by the SFI approach. Hence, relative to the halcyon period of the 1980s, this SFI volume is more modest in its claims, but we think much stronger in its achievements (Blume & Durlauf, 2006).

3.2 The birth of agent-based computer platforms

Crucial for the development of agent-based modeling has been—quite naturally—the increasing availability of computing power[10], which allowed to run even complicated simulations on small PCs[11].

Together with continuous hardware improvements came software development. Aside from traditional programing (e.g. in Fortran, C++, etc.) three different approaches emerged. The first relies on general-purpose mathematical software, like Mathematica, Mathlab or Matcad. The second, exemplified by the Starlogo/Netlogo experience (Resnick, 1994), is based on the idea of an agent-based specific language. The third represents a protocol in the design process, implemented as agent-based specific libraries in standard programing languages (like Java or Python)[12]. The ancestor of these agent-based tools, which was initially developed at the Santa Fe Institute itself, is Swarm (Askenazi *et al.*, 1996). The principles of the Swarm approach are:

- the use of an *object-oriented programing* language, with different objects (and object types) being a natural counterpart for different agents (and agent types);
- a *separate implementation* of the model and the tools used for monitoring and conducting experiments on the model (the so-called 'Observer');
- an architecture that allows nesting models one into another, in order to build a *hierarchy* of 'swarms'—a swarm being a group of objects and a schedule of actions that the objects execute. One swarm can thus contain lower-level swarms whose schedules are integrated into the higher-level schedule.

While in the 'revolutionary' days of the Santa Fe Institute the third approach appeared to be the most promising, a more anarchic attitude (Feyerabend, 1975) has now emerged among practitioners.

Finally, despite the fact that ABMs are most often computer models, and that the methodology could not develop in the absence of cheap and easy-to-handle personal computers, it is beneficial to remember that one of the most well-known ABMs, the pioneering work on spatial segregation by the Nobel laureate Thomas Schelling, did not make use of computers at all (Schelling, 1971). As Schelling recalls, he had the original idea while seated on a plane, and investigated it with paper and pencil. When he arrived home, he explained to his son the rules of the game and got him to move zincs and coppers from the child's own collection on a checkerboard, looking for the results. 'The dynamics were sufficiently intriguing to keep my 12-year-old engaged' (Schelling, 2006).

[10] This is summarized by the empirical 'law' of a twofold increase in performance every 2 years.

[11] It is worth remembering that some of the brightest minds of their time—gathered together around physicists Robert Oppenheimer under the Manhattan project, the World War II U.S. Army project at Los Alamos aimed at developing the atomic bomb—were reported to spend half of their time and effort in order to find smarter algorithms and save precious computing time on the huge but slow machines available (Gleick, 1992).

[12] This allows the possibility to integrate tools developed as separate libraries by third parties (e.g. for graphical visualization, statistical analysis, database management, etc.).

4 Why agents?

Although ACE developed together with the Santa Fe approach, its applicability is by no way limited to the analysis of complex systems. Abstracting from the characteristics of the system being modeled, ACE proves valuable in two cases:

- to get a quick intuition of the dynamics that the system is able to produce, and
- to thoroughly investigate models that are not susceptible of a more traditional analysis, or are susceptible of a more traditional analysis only at too high a cost.

Often, an ABM can be quickly implemented, and can be used not differently from scrap paper. It allows us to experiment with hypothesis and assumptions, and gives a hint to which results can be proved. It often suggests the refinements that might eventually lead to a fully algebraic solution of the model.

However, it might turn out that an analytical solution is not even necessary, or not feasible. Building on Axtell (2000), it is possible to identify three distinct uses of ABMs in the social sciences, apart from the 'scrap paper' one described above. These uses can be ranked according to their auxiliary nature, with respect to analytical modeling[13].

The first use is *numerical computation of analytical models*. Note with Axtell that

> [t]here are a variety of ways in which formal models resist full analysis. Indeed, it is seemingly only in *very* restrictive circumstances that one ever has a model that is completely soluble, in the sense that everything of importance about it can be obtained solely from analytical manipulations.

Situations in which resort to numerical computation may prove useful include (a) when a model is not analytically soluble for some relevant variable, (b) when a model is stochastic, and the empirical distribution of some relevant variable needs to be compared with the theoretical one, of which often few moments are known and (c) when a model is solved for the equilibrium, but the out-of-equilibrium dynamics are not known. In particular, with reference to the last point, it may happen that multiple equilibria exist, that the equilibrium or (at least some of) the equilibria are unstable, that they are realized only in the very long run. Conversely, it may happen that equilibria exist but are not computable[14]. Finally, it may be the case that the equilibrium is less important than the out-of-equilibrium fluctuations or extreme events. Clearly, agent-based simulations are not the only way to perform numerical computations of a given analytical model. However, they may prove effective and simple to implement, especially for models with microfoundations.

The second use is *testing the robustness of analytical models* with respect to departures from some of the assumptions. Assumptions may relate to the behavior of the agents, or to the structure of the model. Note that, in general, as the assumptions are relaxed or altered, an analytical solution becomes very improbable (otherwise, the possibility of changing them could have been easily incorporated in the original work, leading to a more general model). One important feature of ACE is that in considering departures from the assumptions of the reference model, a number of different alternatives can be investigated, thus offering intuition toward a further generalization of the model itself.

The first two uses of ACE models are *complements* to mathematical analysis. The third use is a *substitute*, going beyond the existence of an analytical reference model. It provides *stand-alone simulation models* for (a) problems that are analytically intractable, or (b) problems for which an analytical solution bears no advantage. The latter may happen when negative results are involved, for instance. A simulation may be enough to show that some institution or norm is wrong, or does not work in the intended way. Analytical intractability may arise when more complicated assumptions are needed, or when the researcher wants to investigate the overall effect

[13] The categories identified below correspond only partially to Axtell's.

[14] Axtell (2000) provides references and examples for each case.

of a number of mechanisms (each possibly already analytically understood in simpler models), at work at the same time.

5 The methodological status of agent-based computational economics

A rather common misunderstanding about simulations is that they are not as sound as mathematical models. In particular, they do not offer a compact set of equations—together with their inevitable algebraic solution—which can easily be interpreted and generalized.

In a frequently cited article (Ostrom, 1988), Thomas Ostrom argued that computer simulation is a third symbol system in its own right, aside verbal description and mathematics. This implies that '[s]imulation is neither good nor bad mathematics, but no mathematics at all' (Gilbert & Troitzsch, 1999). Computer simulations are, according to this view, characterized by an intermediate level of abstraction: they are more abstract than verbal descriptions but less abstract than 'pure' mathematics.

Ostrom also argued that '[a]ny theory that can be expressed in either of the first two symbol systems can also be expressed in the third symbol system' (Gilbert & Troitzsch, 1999: 384). This implies that 'there might be verbal theories which cannot be adequately expressed in the second symbol system of mathematics, but can be in the third' (*ibidem*).

This view has become increasingly popular among social simulators themselves, apparently because it offers a shield to the perplexity of the mathematicians, while hinting at a sort of superiority of computer simulations. Our opinion is that both statements are simply and plainly wrong. Simulation *is* mathematics, as we argue in this section. Moreover, the conjecture that any theory can be expressed via simulation is easily contradicted: think for instance at philosophical theories.

Actually, simulations *do* consist of a well-defined (although not concise) set of functions[15]. These functions, which may be either deterministic or stochastic[16], describe a fully recursive system and unambiguously define the macrodynamics of the system. Moreover, the eventual unique equilibrium of the macrodynamics is, in turn, a known function of the structural parameters and initial conditions of the simulation. We will show here that the only difference from a model consisting of an algebraically solved set of equations is in the degree of knowledge that we have about these functions. Let us start from the following general characterization of dynamic micromodels. Assume that at each time t an individual i, $i \in 1, \ldots, n$, is well described by a state variable $x_{i,t} \in \Re^k$. Let the evolution of her state variable be specified by the difference equation:

$$x_{i,t+1} = f_i(x_{i,t}, x_{-i,t}; \alpha_i) \tag{1}$$

where we assume that the behavioral rules[17] may be individual-specific both in the functional form of the phase line $f_i(.)$ and in the parameters α_i, and may also be based on the state x_{-i} of all individuals other than i. Once we have specified the behavior of each individual, we will typically be interested in some macrofeature of our economy, that we may represent as a statistic Y defined over the entire population:

$$Y_t = s(x_{1,t}, \ldots, x_{n,t}) \tag{2}$$

The crucial question now is whether it is possible to solve Equation (2) for each t, regardless of the specification adopted for $f_i(.)$, and the answer is that a solution can always be found by

[15] This section is based on Leombruni and Richiardi (2005). For an advanced mathematical treatment, see Epstein (2006).

[16] In what follows we will refer to the deterministic case. Generalization to the stochastic case requires some changes (mainly regarding the notation), but the idea remains the same.

[17] Here and in the following we use 'behavioral rules' and similar terms in a loose sense that encompasses the actual intentional behaviors of individuals as well as other factors, such as technology, etc.

iteratively solving each term $x_{i,t}$ in (2) using (1):

$$\begin{aligned} Y_0 &= s(x_{1,0}, \ldots, x_{n,0}) \\ Y_1 &= s(x_{1,1}, \ldots, x_{n,1}) \\ &= s(f_1(x_{1,0}, x_{-1,0}; \alpha_1), \ldots, f_n(x_{n,0}, x_{-n,0}; \alpha_n)) \\ &\equiv g_1(x_{1,0}, \ldots, x_{n,0}; \alpha_1, \ldots, \alpha_n) \\ &\vdots \\ Y_t &= g_t(x_{1,0}, \ldots, x_{n,0}; \alpha_1, \ldots, \alpha_n) \end{aligned} \tag{3}$$

The law of motion (3) uniquely relates the value of Y at any time t to the initial conditions of the system and to the values of the parameters α_i. Sometimes[18], g_t may converge to a function not dependent on t[19], so that we also have an expression for the equilibrium value of Y, again as a function of the initial conditions and parameters:

$$Y^e = \lim_{t \to \infty} Y_t \equiv g(x_{1,0}, \ldots, x_{n,0}; \alpha_1, \ldots, \alpha_n) \tag{4}$$

Notice that this formalization describes both 'traditional' dynamic micromodels and agent-based simulations. Indeed, given this common framework, it is easy to discuss the alleged differences in terms of 'mathematical soundness'. To explore this point, let us consider how the framework is implemented in the two approaches. As an example of the 'traditional approach' think of a model based on a representative agent. The behavioral rule (1) will be very simple in structure, since all subscripts i can be dropped, along with any reference to other individuals' behavior. In turn, any 'macro' statistic considered will collapse on a transformation of the state variable of just one individual, and the resulting law of motion (3) will also be very simple. We thus end up with a simple formulation for all Equations (1)–(3), and usually also for Equation (4). By 'simple formulations' we mean that they can be manipulated algebraically, and general propositions about the model can be stated by computing derivatives, comparing different equilibrium solutions, and so on[20].

Let us turn to the agent-based simulation approach. The critical factor rests in the formula for the macrodynamics (3), the law of motion of Y. As t and n increase, the expression for $g_t(.)$ can easily grow enormous, hindering any attempt at symbolic manipulation, that is, any attempt to solve it algebraically[21]. Nevertheless, the functions (3) are completely specified. It is thus possible to explore their local behavior, by computing the value of Y corresponding to different values of the parameters and the initial conditions. A way to extrapolate this point evidence, and thus to recover a local approximation of the shape of $g_t(.)$, is to specify a functional form $\hat{g}_t(x_{1,0}, \ldots, x_{n,0}, \alpha_1, \ldots, \alpha_n, \beta)$ to be fitted on the artificial data generated by the simulation runs, where β's are the coefficients of $\hat{g}_t(.)$. For instance, if $\hat{g}_t(.)$ is assumed to be linear, there will be two coefficients β_0 and β_1 (the intercept and the slope) to be estimated in the artificial data. The use of econometric techniques to approximate $g_t(.)$, starting from a number of—somehow designed—artificial experiments is indeed common practice in the computer science literature. The resulting regression model is also known as *metamodel, response surface, compact model, emulator, etc.* (Kleijnen, 1998).

[18] When the dynamic system has one (or more), stable equilibrium and the initial conditions lie in its (their) basin of attraction.

[19] Or even not dependent on the initial conditions.

[20] Note that the problem of deriving the equilibrium relation (4) from the law of motion (3) is often skipped altogether. Equilibrium conditions are externally imposed, and the dynamics towards the equilibrium is simply ignored: the system 'jumps' to the equilibrium.

[21] This difficulty is the same experienced in game theory models, where games typically become intractable if they involve more than a handful of players.

A particular case of this procedure is to keep fixed all parameters $(x_{1,0}, \ldots, x_{n,0}, \alpha_1, \ldots, \alpha_n, \beta)$ but one at a time, and simply plot Y against this variable, a sort of fully non-parametric estimation of the (partial derivative of) $g_t(.)$. This is called one-at-time (OAT) *sensitivity analysis*. While OAT sensitivity analysis provides a *local exploration* of the dynamics of the system around a (arbitrarily chosen) central configuration, the general procedure described above can in principle be used for a *global exploration* (Leombruni *et al*., 2005).

5.1 Interpretation of the results

A cause of concern with this procedure stems from the possibility that the artificial data may not be representative of all outcomes the model can produce. In other words, it is possible that as soon as we move to different values of the parameters, the behavior of $g_t(.)$ will change dramatically. The metamodel $\hat{g}_t(.)$ will then become a poor description of the simulated world. At a theoretical level, this issue can be answered with two observations. First, if it applies to what we know about the artificial world defined by the simulation model, it also applies to what we know about the real world. As the real data-generating process is itself unknown, stylized facts could in principle go wrong at some point in time. Second, we should not worry too much about the behavior of a model for particular 'evil' combinations of the parameters, so long as these combinations remain extremely rare[22]. If the design of the experiments is sufficiently accurate (often particular combinations of the relevant parameter can be guessed, and oversampled in the artificial experiments), the problem of how 'local' the estimated local data-generating process becomes marginal.

5.2 Estimation

So far we have shown that (i) ABMs are mathematical models and (ii) they can be used to get general results. We will now briefly show that they can also be 'taken to the data', that is, estimated. Estimation means using real data to assign specific values to the structural parameters of the model.

As we have seen, analytical models generally allow us to solve for the function $g_t(.)$: the parameteres can then be estimated in the real data, by means of standard econometric techniques. In a simulation model this cannot be done. However, simulations produce streams of artificial data: to estimate the structural parameters of a simulation all that is needed is to compare these artificial data with the real data. The structural parameters can be changed until the artificial data become as similar as possible to the real data. This strategy is called *indirect inference* (Gourieroux & Monfort, 1997; Smith, 2008). Indirect inference is also applied to analytical models, for example, when it is not possible to write down the likelihood function, or when the function $g_t(.)$ cannot be derived. There are many ways to compare real and artificial data. For instance, simple statistics can be computed both in real and in artificial data, and then aggregated in a unique measure of distance. Clearly, these statistics have to be computed just once in the real data (which does not change), and once every iteration until convergence in the artificial data, which depends on the value of the structural parameters. The change in the value of the parameters of each iteration is determined according to some optimization algorithm, with the aim to minimize the distance. In the *method of simulated moments*, different order of moments are used, and then weighted to take into account their uncertainty (while the uncertainty regarding the simulated moments can be reduced by increasing the number of simulation runs, the uncertainty in the estimation of the real, population moment on the basis of real sample data cannot be avoided).

[22] The relevant exception is when rare events are themselves the focus of the investigation, for instance, as in risk management. Here, simulations may prove extremely useful, by dispensing from making assumptions—such as the Gaussian distribution of some relevant parameters—which may be necessary in order to derive algebraic results but have unpleasant properties—such as excessively thin tails. In a simulation, the reproduction of such rare events is limited only by the computational burden imposed on the computer. However, techniques can be used in order to artificially increase the likelihood of their occurrence.

The intuition behind this is to allow parameters estimated with a higher degree of uncertainty to count less, in the final measure of distance between the real and artificial data (Winker *et al.*, 2007). Having different weights (or no weights at all) impinges on the efficiency of the estimates, not on their consistency, which relies only on ergodicity and stationarity properties that can be checked in the simulated data (Grazzini, 2012). If the number of moments is equal to the number of structural parameters to be estimated, the model is just-identified and the minimized distance, for the estimated values of the parameters, is 0. If the number of moments is higher than the number of parameters, the model is over-identified and the minimized distance is greater than 0. If lower, the model is under-identified. Another strategy is to estimate an *auxiliary model* both in the real and in the artificial data, and then compare the two sets of estimates obtained. The regression coefficients have the same role as the moments in the method of simulated moments: they are just a way of summarizing the data. Hence, if the number of coefficients in the auxiliary model is the same as the number of structural parameters to be estimated, the model is just-identified and the minimized distance is 0. The specification of the auxiliary model is not too important. Of course, one logicl choice for the auxiliary model would be the metamodel $\hat{g}_t(.)$. However, simpler models would also work: it can be proved that misspecification (e.g. omission of a relevant variable in the relationship to be estimated) only affects efficiency, while the estimates of the structural parameters remain consistent (Gourieroux & Monfort, 1997).

6 Conclusions

In a brilliant book (Miller & Page, 2006), John Miller and Scott Page maintain that the interest of many social phenomena lies 'in between' the extremes: in between various scientific fields, in between the few (often just one or two) and the infinitely many agents of neoclassical models, in between the continuous and the discrete, in between the micro and the macro. They argue that the science of complex systems, and in particular the use of computational models, is the most appropriate approach to the investigation of these phenomena.

In this short introduction, we have discussed why this might be the case. We have described the main features of ACE, and showed how it can be a valid methodology for the investigation of social phenomena. The use of ABMs can complement the traditional tools, or can provide a valid alternative. Although the agent-based methodology is used in disciplines as different as biology, medicine, natural resources management, and sociology, its potential for economics is still deeply undervalued (Leombruni and Richiardi, 2005). We therefore conclude with J. Doyne Farmer and Duncan Foley (Farmer & Foley, 2009) that '[t]he economy needs agent-based modelling'. We hope that the initially skeptical or simply unaware reader would, at this point, agree.

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